Limits extra practice

Stearns · same kinds as the Fall limits review · graphing, constants, one-sided limits, discontinuities, infinite limits · new problems

Name _____ Block _____

Show all work. Leave answers in simplest form. Box the final answer. Your own paper is fine. 2 points for work that is organized and readable.

On every graph, draw the asymptote and label it. Mark open circles and closed points. On an infinite limit, state the sign of the numerator and whether the denominator approaches 0 through positive or negative values. One nearby decimal is not a full justification.

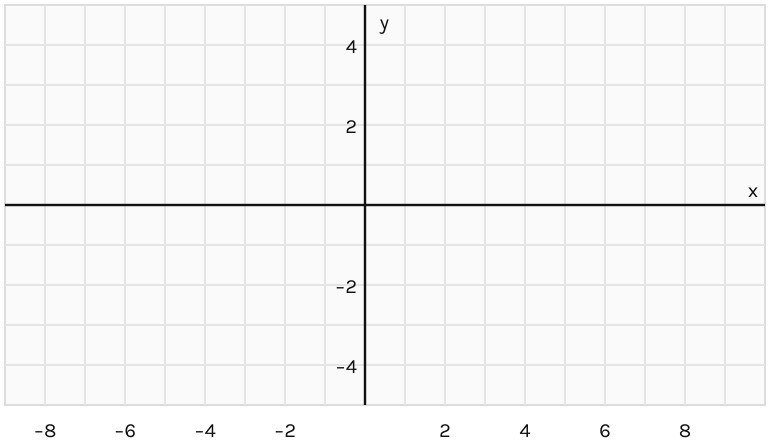
1. Sketch

[5 points]

Using the information below, sketch a possible graph of f. Note that f is a function.

$f(-6) = 1$, $f(0) = f(8) = 2$, $f(3)$ is undefined

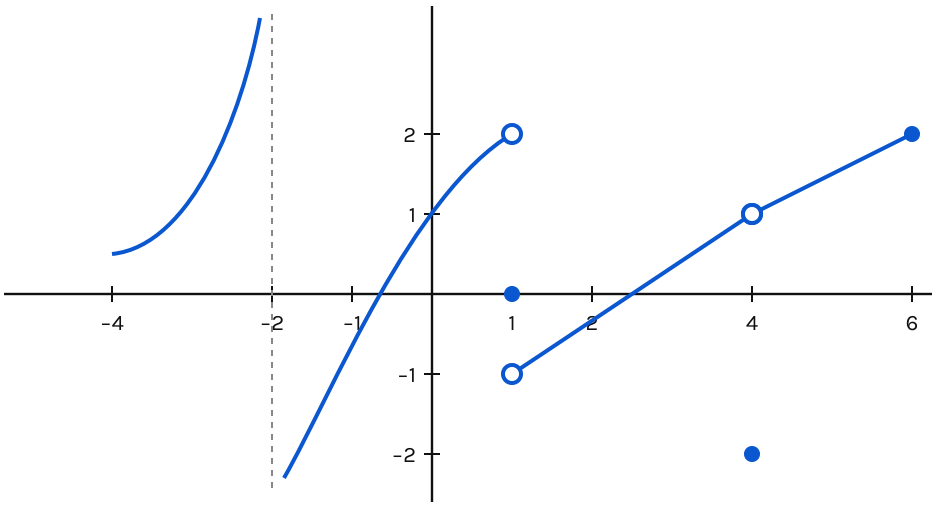
$\lim_{x \to -2^-} f(x) = -\infty$, $\lim_{x \to -2^+} f(x) = +\infty$, $\lim_{x \to 5} f(x)$ = DNE, because the left-hand limit ≠ the right-hand limit



2. Graph

[6 points]

Open circle means the point is not on the graph. Closed circle means it is. The dashed line is a vertical asymptote.



a. $\lim_{x \to -2-} f(x)$, $\lim_{x \to -2+} f(x)$, and $\lim_{x \to -2} f(x)$. Name the discontinuity at $x = -2$.

b. $\lim_{x \to 1-} f(x)$, $\lim_{x \to 1+} f(x)$, $\lim_{x \to 1} f(x)$, and $f(1)$. Name the discontinuity at $x = 1$.

c. $\lim_{x \to 4} f(x)$ and $f(4)$. Name the discontinuity at $x = 4$.

3. One-sided limits

[13 points]

Evaluate. Justify.

a. $\lim_{x \to 5+} \frac{2x - 3}{x - 5}$ [2]

b. $\lim_{x \to -2-} \frac{x - 4}{x + 2}$ [2]

c. $\lim_{x \to 1+} \frac{|x - 1|}{1 - x}$ [3]

d. $\lim_{x \to 0-} \frac{x + 4}{x}$ [2]

e. $p(x) = 2x + 5$ when $x < -1$, and $p(x) = x^2 - 4$ when $x \geq -1$. Find $\lim_{x \to -1-} p(x)$, $\lim_{x \to -1+} p(x)$, $\lim_{x \to -1} p(x)$, and $p(-1)$. Name the discontinuity at $x = -1$. [4]

4. Discontinuities

a. Given $g(x) = \dfrac{x^2 - 4x + 3}{x^3 - 4x^2 + 3x}$, find every x where g is discontinuous. [2]

b. For which of those x-values does lim g(x) exist? [1]

c. $\lim_{x \to -3} f(x)$, where $f(x) = 2x + 1$ when $x \neq -3$, and $f(-3) = 4$. [2]

d. $h(x) = (x^2 - 9)/(x + 3)$ when $x \neq -3$, and $h(-3) = 1$. What kind of discontinuity is at $x = -3$, and what single change makes h continuous there? [2]

5. Constants

The pieces have to agree at the seam. Check both sides, then solve.

a. Find c that makes k continuous for all x.
$k(x) = x^2 - 2x$ when $x < 4$, and $k(x) = cx + 1$ when $x \geq 4$. [3]

b. Find every a that makes f continuous for all x.
$f(x) = x^2$ when $x < a$, and $f(x) = 2x + 3$ when $x \geq a$. [4]

6. Constants in a limit

[4 points]

Find a and b such that

$$\lim_{x \to 0} \frac{\sqrt{(a + bx)} - \sqrt{6}}{x} = \sqrt{6}$$

7. Infinite limits

For a vertical asymptote, state the sign of the numerator and whether the denominator approaches 0 through positive values, negative values, or both.

a. $\lim_{x \to 6^-} \dfrac{x + 2}{x - 6}$ [2]

b. $\lim_{x \to -4^+} \dfrac{3x + 2}{x + 4}$ [2]

c. $\lim_{x \to 1} \dfrac{x + 4}{(x - 1)^2}$ [2]

d. $\lim_{x\to\infty} \frac{4x-1}{2x+5}$ and $\lim_{x\to-\infty} \frac{4x-1}{2x+5}$ [2]

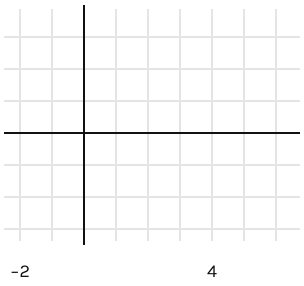
e. $\lim_{x\to\infty} \frac{x^2}{x-2}$ and $\lim_{x\to-\infty} \frac{x^2}{x-2}$ [2]

8. Sketch each statement

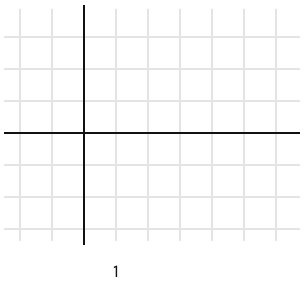
[3 points]

Any correct shape is fine. Each square is 1. The heavy lines are the axes.

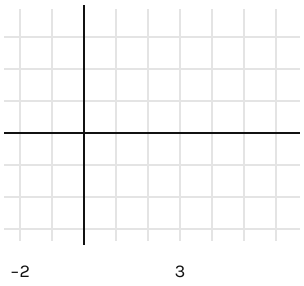
a. $\lim_{x\to4} f(x) = f(4)$



b. $\lim_{x\to1} f(x) = \text{DNE}$



c. $\lim_{x\to-2} f(x) = \lim_{x\to3} f(x)$



Answer key

Check the work, not only the last number. Bold is the answer to box. Any sketch with the listed features is fine.

1. Closed point at (−6, 1). Closed points at (0, 2) and (8, 2). No point at x = 3 (open circle or a gap). Vertical asymptote x = −2, labeled. As x approaches −2 from the left, the graph goes down to −∞. From the right, it goes up to +∞. At x = 5 the left-hand and right-hand limits are different heights, so do not connect across the jump.

2a. Left +∞, right −∞, two-sided limit does not exist. Infinite discontinuity.

2b. Left 2, right −1, two-sided limit does not exist, f(1) = 0. Jump.

2c. Limit 1, f(4) = −2. Removable.

3.

a. **+∞.** Numerator is 7, which is positive. As $x \to 5$ from the right, $x - 5 \to 0$ through positive values.

b. **+∞.** Numerator is negative (about −6). As $x \to -2$ from the left, $x + 2 \to 0$ through negative values. Negative over a small negative is a large positive.

c. **−1.** To the right of 1, $|x - 1| = x - 1$ and $1 - x = -(x - 1)$, so the quotient is −1.

d. **−∞.** Numerator approaches 4, which is positive. As $x \to 0$ from the left, the denominator approaches 0 through negative values.

e. Left 3, right −3, two-sided limit does not exist, p(−1) = −3. Jump.

4a. Factor. Numerator $(x - 1)(x - 3)$. Denominator $x(x - 1)(x - 3)$. Denominator is 0 at x = 0, x = 1, and x = 3. Those are the discontinuities.

4b. $x - 1$ and $x - 3$ both cancel, so the simplified function is $1/x$. The limit exists at x = 1 (value 1) and at x = 3 (value 1/3). At x = 0 the factor does not cancel, so that is a vertical asymptote and the limit does not exist. **x = 1 and x = 3.**

4c. The limit uses the rule for $x \neq -3$, not the listed value 4. $2(-3) + 1 = -5$. **−5.**

4d. $(x - 3)(x + 3)/(x + 3) = x - 3$ for $x \neq -3$. Limit is −6, value is 1, so removable. Set h(−3) = −6.

5a. Left piece at x = 4: $16 - 8 = 8$. Right piece: $4c + 1$. Set them equal: $4c + 1 = 8$, so $c = 7/4$. Check: $k(4) = 4(7/4) + 1 = 8$.

5b. Set $a^2 = 2a + 3$, so $a^2 - 2a - 3 = 0$, which is $(a - 3)(a + 1) = 0$. **a = 3 and a = −1.** Both roots meet. At a = 3, 9 = 9. At a = −1, 1 = 1.

6. Rationalize. Numerator becomes $(a + bx) - 6$, over x times $(\sqrt{(a + bx)} + \sqrt{6})$. If $a \neq 6$, the numerator stays away from 0 as $x \to 0$ and the limit is infinite, not $\sqrt{6}$. So a = 6. Then the expression is $b / (\sqrt{(6 + bx)} + \sqrt{6})$, which approaches $b / (2\sqrt{6})$. Set that equal to $\sqrt{6}$: $b = 2\sqrt{6} \cdot \sqrt{6} = 12$. Check: $12 / (2\sqrt{6}) = \sqrt{6}$. **a = 6, b = 12.**

7a. −∞. Numerator at x = 6 is 8, which is positive. From the left, $x - 6$ approaches 0 through negative values.

7b. −∞. Numerator at x = −4 is −10, which is negative. From the right, $x + 4$ approaches 0 through positive values. A negative over a tiny positive is a large negative.

7c. $+\infty$. Numerator at $x = 1$ is 5, which is positive. $(x - 1)^2$ approaches 0 through positive values from both sides, so the two-sided limit is $+\infty$.

7d. 2 and 2. Divide the top and bottom by x. The pieces with $1/x$ die, and $4/2 = 2$. The degrees match, so both ends do the same thing.

7e. $+\infty$ and $-\infty$. The top has the higher degree, and x^2 stays positive. For large positive x the bottom is positive, so the quotient is a large positive. For large negative x the bottom is negative, so the quotient is a large negative.

8a. A curve that passes through a closed point at $x = 4$. The y-value you approach equals $f(4)$.

8b. A jump or an oscillation at $x = 1$, so the two-sided limit does not exist.

8c. The height approached at $x = -2$ equals the height approached at $x = 3$. The function values at those two x-values do not have to match.