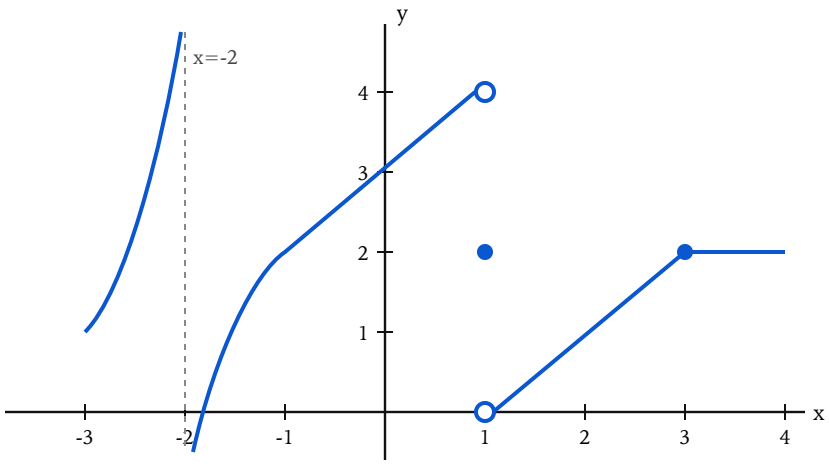
Limits practice

For Stearns, Monday. Covers last year's limits unit: notation, graphs, tables, one-sided limits, continuity, algebra on 0/0, infinite limits. New problems. Calculators: scientific only, same as that test. Show work.

1. Graph

The graph of f is below. Open circle means the point is not on the graph. Closed circle means it is.



1. $\lim_{x \to 1} f(x)$
2. $f(1)$
3. $\lim_{x \to 1+} f(x)$
4. $\lim_{x \to 1-} f(x)$
5. $\lim_{x \to 3} f(x)$
6. $f(3)$
7. $\lim_{x \to -2+} f(x)$
8. $\lim_{x \to -2-} f(x)$
9. $\lim_{x \to -2} f(x)$

2. Table

Values of g near x = -4. Estimate $\lim_{x \to -4} g(x)$. Say if you trust it, and why a table is not a proof.

x	−4.1	−4.01	−4.001	−3.999	−3.99	−3.9
g(x)	0.41	0.0401	0.004001	−0.003999	−0.0399	−0.39

3. Direct substitution

Evaluate. If you can plug in, do that and say why it is allowed.

3. $\lim_{x \to -2} (x^3 - 5x + 1)$

4. $\lim_{t \to \pi/2} (4 \cos t - 1)$ (scientific calculator fine for the cosine)

4. One-sided and piecewise

Let

$p(x) = x^2 - 4x + 7$ if $x < 1$
$p(x) = 6 - 2x$ if $x \geq 1$

5. $\lim_{x \to 1-} p(x)$, $\lim_{x \to 1+} p(x)$, $\lim_{x \to 1} p(x)$, and $p(1)$. Is p continuous at $x = 1$?

Let $q(x) = x / |x|$ for $x \neq 0$, and $q(0) = 0$.

6. $\lim_{x \to 0-} q(x)$, $\lim_{x \to 0+} q(x)$, $\lim_{x \to 0} q(x)$. Name the kind of discontinuity at 0.

5. Indeterminate form

Each of these is 0/0 if you plug in. Simplify, then evaluate. Name the algebra you used.

7. $\lim_{x \to -5} (x^2 + 3x - 10) / (x + 5)$

8. $\lim_{x \to 2} (x^3 - 8) / (x^2 - 4)$

9. $\lim_{h \to 0} (\sqrt{(9+h)} - 3) / h$

10. $\lim_{x \to 0} [\ 1/(x+4) - 1/4\ ] / x$

6. Infinite limits

11. For $r(x) = (2x + 1) / (x - 3)$:
    a. equation of the vertical asymptote
    b. $\lim_{x \to 3+} r(x)$
    c. $\lim_{x \to 3-} r(x)$
    d. $\lim_{x \to 3} r(x)$

12. $\lim_{x \to 0} \sin(1/x)$. Does it exist? Which of the three "limit fails" cases is this?

7. Continuity, in words

13. State the condition, using the limit and the function value, for each:
    a. removable discontinuity
    b. jump discontinuity
    c. infinite discontinuity

14. $c(x) = (x^2 - 16) / (x - 4)$ for $x \neq 4$, and $c(4) = 11$. What kind of discontinuity is at $x = 4$, and what single change makes c continuous there?

8. Short write

15. In three or four lines: when a limit is 0/0, what do you try first, and what do you try if that does not factor?

Answer key

Check the work, not just the number. If your algebra differs and the value matches, that is fine.

1. 1a. does not exist (left is 4, right is 0) 1b. 2 1c. 0 1d. 4 1e. 2 1f. 2 1g. $-\infty$ 1h. $+\infty$ 1i. does not exist (opposite infinities)
2. The values approach 0 from both sides. Estimate 0. A table can miss a hole, a jump, or oscillation between the sample points, so it only suggests the limit.
3. $-8 - (-10) + 1 = 3$. Polynomial, continuous everywhere, so direct substitution is valid.
4. $4(0) - 1 = -1$. Cosine is continuous, so plug in $\pi/2$.
5. Left: $1 - 4 + 7 = 4$. Right: $6 - 2 = 4$. Limit is 4. $p(1) = 4$. Continuous at $x = 1$.
6. Left: -1. Right: 1. Limit does not exist. Jump discontinuity at 0.
7. Factor: $(x+5)(x-2)/(x+5) = x-2$ for $x \neq -5$. Limit is -7. (Factor and cancel.)
8. $x^3-8 = (x-2)(x^2+2x+4)$, $x^2-4 = (x-2)(x+2)$. Cancel $x-2$: $(x^2+2x+4)/(x+2)$ at 2 is $12/4 = 3$. (Sum/difference of cubes, then cancel.)
9. Multiply by the conjugate. Numerator becomes h. Cancel h: $1 / (\sqrt{(9+h)}+3) \to 1/6$. (Rationalize.)
10. Common denominator in the numerator: $[4 - (x+4)] / [4(x+4)]$ all over $x = -x / [4x(x+4)] = -1/[4(x+4)]$ for $x \neq 0$. Limit is -1/16. (Complex fraction.)
11. 11a. $x = 3$ 11b. $+\infty$ (numerator $\to 7$, denominator $\to 0+$) 11c. $-\infty$ 11d. does not exist
12. Does not exist. Oscillating behavior. One-sided limits also fail.
13. 13a. The two-sided limit exists (finite), and either f(a) is undefined or f(a) is not equal to that limit.
    13b. Both one-sided limits exist and are finite, and they are not equal.
    13c. At least one side goes to $\pm\infty$ (unbounded).
14. Limit is 8 (cancel x-4, left with x+4). Function value is 11, so removable. Redefine $c(4) = 8$.
15. A solid answer: plug in first. If you get 0/0, factor and cancel. If it does not factor, rationalize a square root, or combine a complex fraction over a common denominator, then cancel and plug in. If the one-sided limits disagree, stop and say the limit does not exist.