Limits QA practice

Stearns · same shape as the limits review · new problems · scientific calculator

Name _____ Date _____

Show the algebra under the problem, or on your own paper. Do not cram work into the margin. Open circle means the point is not on the graph. Closed circle means it is.

1. Evaluate

If you can plug in, do that and say why. If you get 0/0, simplify first.

1. $\lim_{x \to -1} (2x^3 - x^2 + 4x - 1)$

2. $\lim_{x \to 1} \dfrac{3x^2 + 2}{x^2 + 5x - 2}$

3. $\lim_{x \to -3} \dfrac{x^2 + x - 6}{x + 3}$

4. $\lim_{t \to 4} \dfrac{t^2 - 16}{2t^2 - 5t - 12}$

5. $\lim_{h \to 0} \dfrac{(2 + h)^2 - 4}{h}$

6. $\lim_{x \to 5} \dfrac{\sqrt{(x + 4)} - 3}{x - 5}$

7. $\lim_{x \to -3} \dfrac{\frac{1}{3} + \frac{1}{x}}{x + 3}$

8. $\lim_{x \to 9} \dfrac{3 - \sqrt{x}}{9x - x^2}$

9. $\lim_{h \to 0} \dfrac{\frac{1}{b + h} - \frac{1}{b}}{h}$, with $b \neq 0$

10. $\lim_{x \to 4+} \dfrac{x - 1}{x - 4}$

1. $\lim_{x \to -2^-} \frac{x+5}{x+2}$

2. $\lim_{t \to 1^-} \frac{|t-1|}{t-1}$

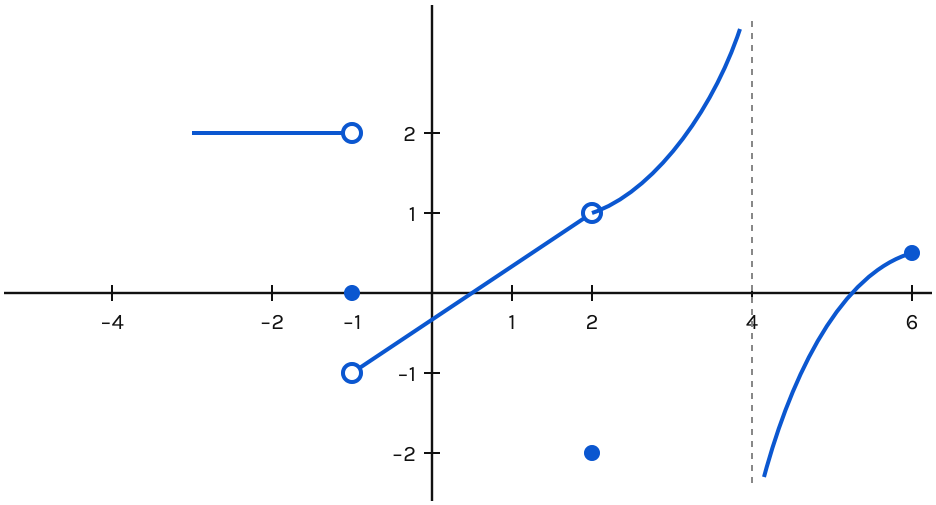
2. Sketch

Sketch a function with all of these. Label the interesting x-values.

$f(2) = 0$, $\lim_{x \to 2^-} f(x) = 3$, $\lim_{x \to 2^+} f(x) = -2$, $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to -1^-} f(x) = -\infty$, $\lim_{x \to -1^+} f(x) = \infty$, $\lim_{x \to \infty} f(x) = 1$

3. Graph

Dashed line is a vertical asymptote.



3. $\lim_{x \to -1^-} f(x)$, $\lim_{x \to -1^+} f(x)$, $\lim_{x \to -1} f(x)$, and $f(-1)$. Name the discontinuity at $x = -1$.

4. $\lim_{x \to 2} f(x)$ and $f(2)$. Name the discontinuity at $x = 2$.

5. $\lim_{x \to 4^-} f(x)$, $\lim_{x \to 4^+} f(x)$, and $\lim_{x \to 4} f(x)$. Name the discontinuity at $x = 4$.

4. Continuous at the point?

16. $f(x) = (x^2 - 9)/(x - 3)$ when $x \neq 3$, and $f(3) = 4$. Is f continuous at $x = 3$? If not, what single change fixes it?

17. $g(x) = x^2 + 1$ when $x < 1$, and $g(x) = 2x$ when $x \geq 1$. Is g continuous at $x = 1$?

5. Constants

The pieces have to agree at the seam. Check both sides, then solve.

18. Find every c that makes this continuous for all x.
    $f(x) = 2x + 1$ when $x \leq c$, and $f(x) = x^2 - 2$ when $x > c$.

19. Find k that makes this continuous for all x.
    $h(x) = kx + 1$ when $x < 3$, and $h(x) = x^2 - 5$ when $x \geq 3$.

6. Build a formula

20. Write a formula with vertical asymptotes $x = -1$ and $x = 2$, and a horizontal asymptote $y = 2$.

21. Write a formula with a removable discontinuity at (3, 7). If you define $f(3) = 2$, is the function continuous there? What value would make it continuous?

7. Tables

22. A coach records a runner's position x in meters at time t in seconds. Estimate $\lim_{t \to 2} x(t)$. What does that number mean about the runner, and what does it not say about x(2)?

t (sec)	1.8	1.9	1.99	2.01	2.1	2.2
x (m)	7.1	7.6	7.94	8.06	8.5	9.1

23. Fill the table with a calculator, then estimate $\lim_{t \to 0+} \sin(t)/t$. Give the estimate, not just the four decimals.

t	0.1	0.01	0.001	0.0001
sin(t)/t				

Answer key

Check the algebra, not only the last number. Several of these have more than one legal formula.

1. −8. Polynomial, so plug in.
2. 5/4. Denominator at x = 1 is 4, not 0, so plug in.
3. −5. Factor $(x + 3)(x - 2)$, cancel $x + 3$.
4. 8/11. Factor $(t - 4)(t + 4)$ over $(2t + 3)(t - 4)$, cancel $t - 4$.
5. 4. Expand: $(4 + 4h + h^2 - 4)/h = 4 + h$.
6. 1/6. Rationalize. Numerator becomes $x - 5$, which cancels.
7. −1/9. Combine the little fractions: $(x + 3)/(3x)$, then cancel $x + 3$, left with $1/(3x)$.
8. 1/54. $3 - \sqrt{x} = (9 - x)/(3 + \sqrt{x})$, and $9x - x^2 = x(9 - x)$. Cancel $9 - x$. Left with $1/(x(3 + \sqrt{x}))$.
9. $-1/b^2$. Common denominator $b(b + h)$, then the h cancels.
10. +∞. Numerator is 3, denominator is a tiny positive number.
11. −∞. Numerator is 3, denominator is a tiny negative number.
12. −1. To the left of 1, $t - 1$ is negative, so the absolute value flips the sign.

Sketch. A jump at x = 2 (closed dot on the axis, left height 3, right height −2). Vertical asymptote at x = −1, down on the left and up on the right. The right end levels off toward y = 1, and the left end climbs as x goes to −∞. Any sketch with those features is fine.

13. Left 2, right −1, two-sided limit does not exist, f(−1) = 0. Jump.
14. Limit 1, f(2) = −2. Removable.
15. Left +∞, right −∞, two-sided limit does not exist. Infinite discontinuity.
16. No. Limit is 6, value is 4. Set f(3) = 6.
17. Yes. Left 2, right 2, g(1) = 2.
18. c = 3 and c = −1. Set $2c + 1 = c^2 - 2$, so $c^2 - 2c - 3 = 0$, which is $(c - 3)(c + 1) = 0$. Both roots meet.
19. k = 1. At x = 3, $3k + 1 = 4$.
20. One answer: $2x^2 / (x^2 - x - 2)$. Denominator is $(x - 2)(x + 1)$. Leading terms give y = 2.
21. One answer: $(x^2 + x - 12)/(x - 3)$, which simplifies to $x + 4$ for $x \neq 3$, so the hole is (3, 7). Defining f(3) = 2 keeps it discontinuous. Set f(3) = 7 to make it continuous.
22. About 8 meters. The position is approaching 8 m as time approaches 2 s. The table does not say the runner is at 8 m at the instant t = 2.
23. About 0.9983, 0.99998, 1.0000, 1.0000. Estimate 1.